A concept we have not yet covered in this series is factorials (though we used some factorials in the post Power in Factorials). Let's first discuss the basics of factorials. Once we do, we will see that most factorial expressions can be easily solved using a single method: taking common!

First of all, what is (n!)?

n! = 1*2*3*4*5*6*…*(n – 2)*(n -1)*n

Let's take some examples:

0! = 1 (mind you, it is not 0)

1! = 1

2! = 1*2

3! = 1*2*3

4! = 1*2*3*4

and so on…

Look carefully. Do you see any relation between 3! and 4!? Sure. 3! Appears in 4! too.

4! = 1*2*3*4 = (1*2*3)*4 = (3!)*4

Similarly, 2! is also a part of 3! as well as 4!

4! = 1*2*3*4 = (2!)*3*4 = (3!)*4

As a general note, we can say that:

n! = (n – 1)! * n

n! = (n – 2)! * (n – 1) * n

n! = (n – 3)! * (n – 2) * (n – 1) * n

and so on…

We can write n! in many different ways. We use whatever suits us best in the question. How does knowing this help us solve questions? Let's see:

Question: If (n-2)!= [n! + (n-1)!]/99 and n is a positive integer, how many distinct values can n take?

(A) 0
(B) 1
(C) 2
(D) 3
(E) Infinite

Solution: We need to solve this equation to find out the values of x which satisfy it. But how do we solve equations with factorials in them?

(n-2)!= [n! + (n-1)!]/99

It looks rather complicated, right? It is not, actually! Let's use what we have just learned. We need to separate the factorials from the rest of the equation. To do that, we need to take something common. The left hand side has (n-2)!

We know that n! = (n-2)!*(n-1)*n

and (n-1)! = (n-2)!*(n-1)

The equation becomes: (n-2)!= [(n-2)!*(n-1)*n + (n-2)!*(n-1)]/99

(n-2)! = (n-2)!*[(n-1)*n + (n-1)]/99

99(n-2)! = (n-2)!*[(n-1)*n + (n-1)]

(n-2)!*[(n-1)*n + (n-1) - 99] = 0

The product of two factors (n-2)! and [(n-1)*n + (n-1) - 99] must be 0 so at least one of them must be 0. Notice that factorial of a number cannot be 0 so the other factor i.e. [(n-1)*n + (n-1) - 99] must be 0.

[(n-1)*n + (n-1) - 99] = 0

n^2 = 100

n can take two values: 10 and – 10

But it is given to us that n is a positive integer so only 10 is acceptable.

Hence, there is only 1 value which satisfies this equation.

Answer (B)

Remember, when dealing with multiple factorials, all you can do is take something common. But then, that may be all you need to do!